Scaling Properties of Traffic in Communication Networks

Probabilistic Resources Allocation in Cloud Environments

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Historical perspective

1917 Erlang: circuit switching networks
1968-69 exponential
  Poisson
  Markov
1969 Kleinrock: packet switching networks
  Markov
1988 Van Jacobson: AIMD - TCP
1993 Leland Willinger: LRD LAN
1994 Paxson Floyd: LRD WAN
1997 Crovella: heavy tails
1997 Taqqu: ON/OFF model
1992 Tim Berners Lee: Web
1994 Norros: queues and LRD
1997 Park Crovella: QoS degradation
1998 Padhye: Markov - 1 TCP source
2004 Roberts: QoS insensitivity
2004 Mandjes: QoS ON/OFF (theo.)
1997 Taqqu: LRD LAN

Some open questions:
- Long Range Dependence / Heavy Tailed distributions impact on QoS?
- Existing models (e.g. Padhye) only predict mean metrics (e.g. throughput): what about variability?
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exponential
Poisson
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1968-69
Mandelbrot
LRD (fBm)
heavy tails

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Our approach

To combine *theoretical models* with *controlled experiments* in realistic environments and *real-world traffic traces*
Congestion essentially arises at the access points
→  Simplified System: single bottleneck

- Users’ behavior: ON/OFF source model

- *MetroFlux*: a probe for traffic capture at packet level (O. Goga,...)
Long memory in aggregated traffic: the Taqqu model

- Heavy-tailed distributed ON periods: heavy tail index $\alpha_{ON} > 1$

Theorem (Taqqu, Willinger, Sherman, 1997)

*In the limit of a large number of sources $N_{src}$, if:*
- flow throughput is constant,
- same throughput for all flows;
*aggregated bandwidth $B^{(\Delta)}(t)$ is long range dependent, with parameter:*

$$H = \max \left( \frac{3 - \alpha_{ON}}{2}, \frac{1}{2} \right)$$

**Long memory: long range correlation ($H > 1/2$)**

$$\text{Cov}_{B^{(\Delta)}}(\tau) = \mathbb{E} \left\{ B^{(\Delta)}(t)B^{(\Delta)}(t + \tau) \right\} \sim \tau^{2H - 2}$$

Variance grows faster than $\Delta$: $\text{Var} \left\{ B^{(\Delta)}(t) \right\} \sim \Delta^{2H}$
Theorem validation on a realistic environment

- Controlled experiment: *MetroFlux* 1 Gbps, 100 sources, 8 hours traffic
- UDP/TCP: throughput limited to 5 Mbps (no congestion)

$$\alpha_{ON} = 1.5$$

⇒ Protocol has no influence at large scales
⇒ Long memory shows up beyond scale $\Delta = \mu_{ON}$ (mean flow duration)
Influence of flow mean throughput / duration correlation

- Web traffic acquired at in2p3 (Lyon) with MetroFlux 10 Gbps

ON Distribution

Size Distribution

- Heavy-tailed ON periods, $\alpha_{\text{ON}} = 1.2$
- Heavy tailed flow sizes, $\alpha_{\text{SI}} = 0.85$
Influence of flow mean throughput / duration correlation

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**ON Distribution**

- Heavy-tailed ON periods, $\alpha_{ON} = 1.2$

**Size Distribution**

- Heavy tailed flow sizes, $\alpha_{SI} = 0.85$

**Mean throughput**

- Flow throughput and duration are correlated:

  \[
  \mathbb{E}\{\text{thr.}|\text{dur.}\} \propto (\text{dur.})^{\beta-1}, \quad \beta = \frac{\alpha_{ON}}{\alpha_{SI}} \ (= 1.4)
  \]

  ⇒ Which heavy tail index does control LRD? ($\alpha_{ON}$, $\alpha_{SI}$)?
Taqqu model extension

- Planar Poisson process to describe arrival instant vs duration

Log-diagram, $\beta > 1$

$$\log \text{Var} \{ B(\Delta) \} \text{ scale } \Delta$$
Planar Poisson process to describe arrival instant vs duration

**Proposition (LGVBP, 2009)**

**Model:** \( \mathbb{E}\{\text{through.}|\text{dur.}\} = M \cdot (\text{dur.})^{\beta - 1} \); \( \text{Var}\{\text{through.}|\text{dur.}\} = V \)

\[
\text{Cov}_{B(\Delta)}(\tau) = CM^2 \tau^{-(\alpha ON - 2(\beta - 1)) + 1} + C' V \tau^{-\alpha ON + 1}
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Log-diagram, \( \beta > 1 \)

- threshold \( \tau^* = \left( \frac{C' V}{CM^2} \right)^{1/(2(\beta-1))} \)
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\[ \rightarrow \text{if } \Delta \gg \tau^*: \quad H = H_{\text{Taqqu}} + (\beta - 1) \]
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  - if \( \Delta \gg \tau^* \): \( H = H_{\text{Taqqu}} + (\beta - 1) \)
  - if \( \Delta \ll \tau^* \): \( H = H_{\text{Taqqu}} \)
Taqqu model extension

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\[ \mathbb{E}\{ \text{through.}|\text{dur.}\} = M \cdot (\text{dur.})^{\beta - 1}; \quad \text{Var}\{ \text{through.}|\text{dur.}\} = V \]

\[ \text{Cov}_{B(\Delta)}(\tau) = CM^2 \tau^{-(\alpha\text{ON}-2(\beta-1))+1} + C' V \tau^{-\alpha\text{ON}+1} \]

Log-diagram, \( \beta > 1 \)

- Correlations intensify LRD \( (\beta > 1) \)
- Traffic evolution, future Internet: “flow-aware” control mechanisms, FTTH
LRD impact on QoS: a brief (experimental) outlook

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- Negative on finite queues with UDP flows [cf. Mandjes, 2004 (infinite queues)]
  - LRD degrades QoS for large queue sizes (beyond some threshold)
  - but the threshold depends on the considered QoS metric (loss rate vs mean load)

  - LRD has contradictory effects on QoS metrics depending on:
    - with slow start without slow start
    - Delay ↘ ↗
    - loss rate ↘ →
    - mean throughput → ↗
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<table>
<thead>
<tr>
<th></th>
<th>with slow start</th>
<th>without slow start</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay</td>
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    |-----------------|--------------------|
    | Delay ↓         | ↑                  |
    | loss rate ↓     | →                  |
    | mean throughput → | ↑                |

- Heavy tailed distributions (i.e LRD) can favour QoS for large flows

- But in general, QoS is a complex function of multiple variables
Second level of description: single TCP source traffic

Sources

\[ N_{\text{src}} \]

Agrégat

\[ \tau_{\text{ON}} \quad \tau_{\text{OFF}} \]
Second level of description: single TCP source traffic

- \( W_i \) single TCP source traffic detail
- Long-lived flow → stationary regime

⇒ How to characterize the congestion window evolution?
Markov model

- long-lived flow stationary regime: AIMD
- model: \((W_i)_{i \geq 1}\) finite Markov chain (irreducible, aperiodic), transition matrix \(Q\):
  \[
  \begin{align*}
  Q_{w, \min(w+1, w_{\max})} &= 1 - p(w), \\
  Q_{w, \max(\lfloor w/2 \rfloor, 1)} &= p(w).
  \end{align*}
  \]
- \(p(\cdot)\) loss probability of at least one packet, only depends on the current congestion window (hyp.)
- Example: [Padhye, 1998] Bernoulli loss: \(p(w) = 1 - (1 - p_{\text{pkt}})^w\)
Almost sure mean throughput

\[ W_i \text{ (paquets)} \]

\[ \overline{W}^{(n)} \]

\[ n \rightarrow \infty \]

\[ i \text{ (RTT)} \]

- mean throughput at scale \( n \) (RTT):
  \[ \overline{W}^{(n)} = \frac{1}{n} \sum_{i=1}^{n} W_i \]

Ergodic Birkhoff theorem (1931): almost sure mean

For almost all realisation, the mean throughput at scale \( n \) converges towards a value corresponding to the expectation of the invariant distribution:

\[ \overline{W}^{(n)} \xrightarrow{p.s.} \overline{W}^{(\infty)} = \mathbb{E}\{W_i\} \]

- Example: [Padhye, 1998],
  \[ \overline{W}^{(\infty)} \xrightarrow{p.pkt \rightarrow 0, RTT=1, MSS=1} 0 \sqrt{\frac{3}{2p_{pkt}}} \]
Throughput variability: Large Deviations

- \( \overline{W}^{(n)} \approx \alpha \neq \overline{W}^{(\infty)} \) Rare events

Large Deviations theorem (Ellis, 84)

\[ \mathbb{P}(\overline{W}^{(n)} \approx \alpha) \sim \exp(n \cdot f(\alpha)) \]

- \( f(\alpha) \) Large Deviation spectrum
- \( \rightarrow \) Scale invariant quantity
Throughput variability: Large Deviations

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\[ f(\alpha) \]

\( \Rightarrow \) Does a similar theorem exist for a single realization?
**Large Deviation on almost all realizations**

\[
W_i \quad \text{intervalle 1} \\
\overline{W}_1^{(n)} \\
\text{intervalle } k_n \\
\overline{W}_{k_n}^{(n)} \\
\text{intervalle } k_n \\
\text{i}
\]

**Large Deviation theorem on almost all realisations (Loiseau et al., 2010)**

For a given \( \alpha \), if \( k_n \geq e^{nR(\alpha)} \), then a.s.

\[
\frac{\# \left\{ j \in \{1, \cdots, k_n\} : \overline{W}_j^{(n)} \simeq \alpha \right\}}{k_n} \xrightarrow[n \to \infty]{\sim} \exp(n \cdot f(\alpha))
\]

- “Price to pay”: exponential increase of the number of intervals
- Finite realization (of size \( N \)): \( nk_n = N \)

\[ [\alpha_{\min}(n), \alpha_{\max}(n)] \text{ support of observable spectrum at scale } n \]
Large Deviation on almost all realizations

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- "Price to pay": exponential increase of the number of intervals
- Finite realization (of size $N$): $nk_n = N$
  $$\Rightarrow [\alpha_{\min}(n), \alpha_{\max}(n)] \text{ support of observable spectrum at scale } n$$
- Theory: $p(\cdot) \rightarrow Q \rightarrow f(\alpha), R(\alpha), \alpha_{\min}, \alpha_{\max}$
- Practice: $(W_i)_{i \leq N} \rightarrow$ observed distribution
Results: example of Bernoulli losses \( (p_{\text{pkt}} = 0.02) \)
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- Apex: almost sure mean: 8.6 packets (Padhye: \( \sqrt{\frac{3}{2p_{\text{pkt}}}} = 8.66 \)
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- Superimposition at different scales → scale invariance
Results: example of Bernoulli losses ($p_{\text{pkt}} = 0.02$)

- Apex: almost sure mean: 8.6 packets (Padhye: $\sqrt{\frac{3}{2p_{\text{pkt}}}} = 8.66$)
- Superimposition at different scales $\rightarrow$ scale invariance
- Beyond $n = 100$: variability
  - $n = 100$, portion of intervals with mean $\sim 11$: $e^{-100 \times 0.01} = 0.37$
  - $n = 200$, portion of intervals with mean $\sim 11$: $e^{-200 \times 0.01} = 0.14$
  $\Rightarrow$ More accurate information than the almost sure mean
Results II: case of a long-lived flow

- losses: not Bernoulli
- empirical losses

\[ p(w) \]

\[ f(\alpha) \]

\[ \alpha_{\min}, \alpha_{\max} \]

\[ n (RTT) \]
Two important assets for Large Deviations Utility

**General result** ("Large deviations for the local fluctuations of random walks", J. Barral, P. Loiseau, *Stochastic Processes and their Applications*, 2011)

A wide class of processes (stationary & mixing) verifies an empirical large deviation principle. In particular, this result holds true any time series that can reliably be modelled by an irreducible, aperiodic Markov process.


We derived a consistent estimator of the large deviation spectrum from a finite size time series (observation samples). We proved convergence on mathematical objects with scale invariance properties (multifractal measures and processes).

Empirical estimation from a finite length trace
Probabilistic Resources Allocation Based on a LDP

Workload Volatility

**Context**
Applications that undergo highly time-varying (elastic) workloads (e.g. Buzz demand in a VoD system)
Probabilistic Resources Allocation Based on a LDP

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Number of current VoD users

![Graph showing number of current VoD users over time with under-provisioning highlighted](image-url)
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**Approach**  Combine the three ingredients:

- A sensible (epidemic) model to catch the burstiness and the dynamics of the workload
- A (Markov) model that verifies a large deviations principle
- A probabilistic management policy based on the large deviation characterisation
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An epidemic based model for volatile workload

A hidden state Markov process with memory effect [IEICE 2012, TRAC 2013]

\[ S_{\text{Susceptible}} \]  \( \rightarrow \)  \[ I_{\text{Infected}} \]  \( \rightarrow \)  \[ R_{\text{Recovered}} \]

\( S \): current # of viewers
\( I \): current # of infected
\( R \): past-viewers

Infinite population  \( I(t) \): workload time series  \( R(t) \): information diffusers
An epidemic based model for volatile workload

A hidden state Markov process with memory effect [IEICE 2012, TRAC 2013]

\[ S_{\text{Susceptible}} \rightarrow I_{\text{Infected}} \rightarrow R_{\text{Recovered}} \]

\( i, r \rightarrow i+1, r \leftarrow i-1, r+1 \)

\( \beta(i+r) + l \quad \mu r \quad \gamma i \quad a_1 \quad a_2 \)

\( i : \text{current \# of viewers} \quad r : \text{current \# of infected} \)
Model identification and evaluation

A. MCMC based estimation procedure for the model's parameters [Gretsi 2013]

Param. estimation precision

\[ \hat{\beta}_1, \hat{\beta}_2, \hat{\gamma}, \hat{\mu}, \hat{\lambda}_1, \hat{\lambda}_2 \]
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VoD workload - trace I
Model identification and evaluation

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Proposed Model
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MMPP/M/1
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P. Gonçalves (Inria)
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Autocorrelation function

VoD workload - trace I

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MMPP/M/1
Large Deviations Principle

A process $l_t$ verifies a large deviations principle:

$$\mathbb{P}\{\langle l_t \rangle_{\tau} \in [\alpha - \varepsilon_{\tau}, \alpha + \varepsilon_{\tau}]\} \sim \exp(\tau \cdot f(\alpha)), \quad \tau \to \infty$$

- $\tau$ : average time scale
- $f(\alpha)$ : large deviations spectrum of $l_t$
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"Dynamic" implies time scale: a notion that is explicit in large deviations principle
Overflow probability

\[ \tau = \alpha \exp(\tau \cdot f(\alpha)) \]

\[ P(\alpha') \]

\[ \text{time} \]

\[ 0 \quad 10 \quad 20 \quad 30 \]

\[ 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \]

\[ \text{Overflow probability} \]
Overflow probability

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Overflow probability

\[ \tau = \alpha \exp(\tau \cdot f(\alpha)) \]

\[ <l>_{\tau} = \alpha \]

Probability of \( \alpha \)
Overflow probability

\[ \exp(\tau . f(\alpha)) \]

\[ \langle I \rangle_{\tau = \alpha} \]

\[ \text{'Probability of } \alpha \text{'} \]
Overflow probability

\[ \tau = \alpha \exp(\tau f(\alpha)) \]

'Probability of $\alpha$'

\[ \langle l \rangle_{\tau} = \alpha \]

Scaling Properties of Traffic

Large Deviations for probabilistic resource management

P. Gonçalves (Inria)
Probabilistic resource provisioning

Probability of mean workloads

\[ \tau < \tau \]
Probabilistic resource provisioning

\[ \tau < \tau \]

\[ P\{ <l> > \delta \} \sim 10\% \]

\[ P\{ <l> > \delta \} \sim 1\% \]
Probabilistic resource provisioning

Resource provisioning based on a time scale dependent performance evaluation
Probabilistic resource provisioning

Resource provisioning based on a time scale dependent performance evaluation

→ Dynamic management
Optimal reactive time scale for reconfiguration
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Reactivity scale for reconfiguring resource allocation is a compromise between:

- the level of congestion (or losses) yielding tolerable performance degradation
- the affordable price for a frequent reconfiguration of infrastructures
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Assume admissible bounds for these 2 competing factors:

\[ \alpha^* > \alpha_{a.s.} \]

beyond, it is mandatory (or profitable) reallocationg resources

\[ \leftarrow \text{capex performance concern} \]

\[ \sigma^* \]

acceptable probability of occurrence of overflows

\[ \leftarrow \text{opex cost} \]
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and $f(\alpha)$ is identifiable
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Optimal reconfiguration time scale for dynamic resource provisioning:

\[ \tau^* : \Pr\{\langle I \rangle_{\tau^*} \geq \alpha^* \} \approx \int_{\alpha^*}^{\infty} P_{\tau^*}(\alpha) \, d\alpha > \sigma^* \]
Elastic link capacity dimensioning
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The Service Level Agreement fixes:

- ... 
- an admissible level of losses due to link congestion
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$C_0 = \alpha_{a.s.}$ The dedicated link capacity (nominal functioning)
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\[
C_0 = \alpha_{a.s.} \quad \text{The dedicated link capacity (nominal functioning)}
\]

\[
\tilde{C}_{\tau_{\min}} \quad \text{The shared bandwidth needed to absorb bursty overflows, while guaranteeing QoS (loss rate) conformed to SLA:}
\]

\[
\tilde{C}_{\tau_{\min}} = \int_{\alpha_{a.s.}}^{\infty} (\alpha - \alpha_{a.s.})P_{\tau_{\min}}(\alpha) \, d\alpha
\]
Elastic link capacity dimensioning

The Service Level Agreement fixes:

- ... 
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\[ \tilde{C}_{\tau_{\text{min}}} \quad \text{The shared bandwidth needed to absorb bursty overflows,} \]
\[ \quad \text{while guaranteeing QoS (loss rate) conformed to SLA:} \]
\[ \tilde{C}_{\tau_{\text{min}}} = \int_{\alpha_{a.s.}}^{\infty} (\alpha - \alpha_{a.s.}) P_{\tau_{\text{min}}} (\alpha) \, d\alpha \]

$\tau_{\text{min}}$ Determined by the buffer size provisioned to dampen traffic volatility
Concluding remarks

Scaling laws present in many (complex) systems are likely to become ever more ubiquitous (big data sets, heterogeneity, traffic awareness...). Impact on performance is still little known. Large Deviation Principle is insufficiently exploited so far. It holds true for a large class of modelling processes and takes explicitly into account the role of time scale. It conveys information about the dynamics of the process.
## Concluding remarks

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